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Steady state self-focusing of laser beam in an axially inhomogeneous plasma with collisional non-linearity

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Abstract In the present paper, we have investigated the self-focusing behaviour of radially symmetrical Gaussian laser beam propagating in an axially inhomogeneous plasma. Considering the non-linearity to arise from collisional heating and following the extended version of Sodha *et al* theory based on the WKB and paraxial-ray approximation the self-focusing behaviour has been investigated in some detail The effect of different type of axial inhomogeneities in plasma, on the self-focusing of laser beam have been studied for arbitrary large magnitude of non-linearity Results indicate that the plasma behaves as an oscillatory wave-guide The self-focusing is found to depend on type of axial inhomogeneity as well as characteristics scale length of axial inhomogeneity In case of linearly increasing and parabolic convex type axially inhomogeneous plasmas, observed oscillatory behaviour appears to be damped which is not due to any kind of loss mechanism but correlated to the variation in charge density

Keywords Laser-matter interaction, non-linear phenomena, self-focusing

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1. Introduction

The self-focusing of laser beam in non-linear medium has been a subject of many theoretical, numerical as well as experimental investigations [1–3]. But most of these studies (theoretical and numerical) are limited to various approximations such as homogeneous medium, non-linear part of the dielectric constant much smaller than the linear part *etc.* These approximations are rather restrictive and limit the applicability of the theory to many real life situations Results of these studies, in many cases, are far off from the observed experimental observations [4,5].

Transmission of an intense light beam through an inhomogeneous non-linear wave guide has a wide variety of potential applications [6] such as in integrated optics (four wave mixing, optical bi-stable devices), medical procedure (surgery, cauterisation), industrial processes (cutting, welding *etc.*) and power transmission (for electric hazard zones).

Sodha and coworkers [7,8] had developed a steady state paraxial theory of self-focusing of laser beam in a non-linear, non-absorbing homogeneous medium. One of the important feature of their theory is that non-linearity is arbitrarily large as observed in many of the real life situations. The extended version of this theory has been used in the present study of self-focusing of laser beam in inhomogeneous plasma medium. This study is restricted to the Gaussian beam in linearly increasing and decreasing, exponentially varying as well as in parabolic type inhomogeneous plasma. To compute the results, non-linear mechanism considered in the present study is collisional heating.

2. Inhomogeneous plasma medium

Inhomogeneous plasma means that charge density is not uniform throughout the space where laser plasma interactions are considered. In real situation, this variation is quite complex and depends upon the type of plasma; man made or nature produced (ionospheric *etc.*). For the study of self-focusing of laser beam in plasma, some simple models for variations of charge density are devised and considered here in the present analysis. The inhomogeneity in charge density of the plasma at any time in space can be represented by the relation [9]

$$N(x,y,z,t) = N_0 W(x,y,z,t), \tag{1}$$

where N_0 is the density of the plasma at $x = 0$, $y = 0$, $z = 0$ and $t = 0$. Here, $W(x, y, z, t)$ is the density profile function and may have different shapes for different types of inhomogeneities.

Let the electromagnetic wave whose effect is to be studied, is propagating in z-direction in plasma. In axially inhomogeneous plasma, the electron density varies along the z-direction only *i.e.* the non-uniformity in charge density is present in the propagation direction only. The variation in charge density in the x–y plane is not present there and system is supposed to be under steady-state *i.e.* time-independent. For such type of inhomogeneity (axial only), the eq. (1) can be rewritten as

$$N(z) = N_0 W(z), \tag{2}$$

where N_0 is a constant (density of plasma medium at the boundary where wave is incident on it *i.e.* at $z = 0$) and density profile function $W(z)$ is only z-dependent. This function $W(z)$ can have different shapes corresponding to different types of axially inhomogeneous plasma. In the present study, few shapes are considered which are founded to be of practical importance.

(a) Linearly increasing axial inhomogeneity :

The charge density is supposed to increase linearly with the propagation distance. For such type of axially inhomogeneous plasma, density profile function which is of practical importance can be written as

$$W(z) = 1 + z/L. \tag{3}$$

Here, z is the propagation distance in the plasma medium and L is the characteristics scale length of axial inhomogeneity.

(b) Linearly decreasing axial inhomogeneity

In case of model suggested for laser induced fusion technology–a plasma layer of non-uniform charge density is usually generated due to intense laser pulse interaction with dielectric target. For such type of inhomogeneous plasma slab, charge density decreases linearly with the distance from the slab as

$$W(z) = 1 - z/L. \tag{4}$$

Here $z < L$, otherwise eq. (4) leads to purely imaginary plasma frequency and unrealistic situation.

(c) Exponentially varying axial inhomogeneities :

The electron charge density functions for such type of axially inhomogeneous plasma which are considered in the present study, can be written as

$$W(z) = 1 + \frac{z^2}{L^2} \exp\left(-\frac{z^2}{L^2}\right) \tag{5a}$$

and $$W(z) = 1 + \exp\left(1 - z/L\right)^2. \tag{5b}$$

(d) Parabolic inhomogeneities :

Electron charge density profile for such type of axially inhomogeneous plasma can be written as

$$W(z) = 1 - \sigma\frac{z^2}{L^2}, \tag{6}$$

where $\sigma = \pm 1$, corresponds to convex or concave shape for density profile.

When $\sigma = -1$, then from above equation, one gets

$$W(z) = 1 + \frac{z^2}{L^2} \quad \text{(Convex)} \tag{6a}$$

and when $\sigma = +1$,

$$W(z) = 1 - \frac{z^2}{L^2} \quad \text{(Concave)}. \tag{6b}$$

Here $z < L$, otherwise eq. (6b) will lead to unrealistic situation.

Variation of $W(z)$ with z for different type of density profiles are shown in Figure 1.

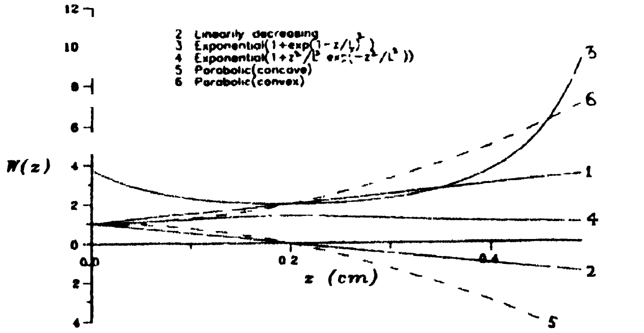

Figure 1. Plots of electron density profile function $W(z)$ for different types of axially inhomogeneous plasmas against propagation distance (z). Here $L = 0.2$ cm

The value of plasma frequency depends on the plasma charge density. Therefore, it is noticed that the plasma frequency is not a constant (as in case of homogeneous plasma) but varies in inhomogeneous plasma as

$$\omega_p^2 = \frac{4\pi N e^2}{m}.$$

Substitution of N from eq. (2) for axially inhomogeneous plasma, gives

$$\omega_p^2 = \frac{4\pi N_0 e^2 W(z)}{m}$$
$$= \omega_{p0}^2 W(z), \tag{7}$$

where $\omega_{p0}^2 = \dfrac{4\pi N_0 e^2}{m}$ is the homogeneous plasma frequency or the plasma frequency at the boundary of inhomogeneous plasma. Eq. (7) gives the z-dependence of plasma frequency in case of axially inhomogeneous plasma. For different shapes of $W(z)$ *i.e.* for different type of inhomogeneities, this dependence is going to be different.

3. Self-focusing equation with arbitrary large non-linearity

The intensity distribution of a linearly polarised Gaussian laser beam can be written as

$$EE^* = E_0^2 \exp(-r^2 / r_0^2), \tag{8a}$$

where r is the radial coordinate of the cylindrical coordinate system and r_0 is the intital beam width. E_0 represents the amplitude of the electric field due to propagating laser beam.

For the study of self-focusing phenomena, the non-linear dielectric constant of the medium can be written as [10,11]

$$\epsilon(\langle EE^* \rangle) = \epsilon_0 + \Phi(\langle EE^* \rangle). \tag{8b}$$

In the paraxial-ray approximation, one generally expands Φ around $\Phi \cong 0$. However with such an expansion one can study only those cases where $\Phi << \epsilon_0$. To sutdy self-focusing for arbitrary large non-linearity, one should expand Φ around an arbitrary large value at $r = 0$. In order to do this, the non-linear dielectric constant of the medium may be rewritten as [10]

$$\epsilon[< EE^* >] = \epsilon_0 + \frac{k(0)E_0^2}{2k(f)f^2}$$

$$+ \Phi[< EE^* >] - \Phi\left[\left\langle \frac{k(0)E_0^2}{2k(f)f^2}\right.\right.$$

or, $$\epsilon(< EE^* >) = \epsilon_0'(f) + \psi(r,f), \tag{9a}$$

where $$\epsilon_0'(f) = \epsilon_0 + \Phi\left[\left\langle \frac{k(0)E_0^2}{2k(f)f^2}\right.\right. \tag{9b}$$

$$\psi(r,f) = \Phi[\langle EE\rangle] - \Phi\left|\left\langle \frac{k(0)E_0^2}{2k(f)f^2}\right\rangle\right| << \epsilon_0'(f). \tag{9c}$$

Here, f is the dimensionless beam-width parameter, defined below in eq. (15) and k is the propagation constant defined below in eq. (11). Here, in eq. (9), < > represents time average of many cycles. Using the WKB approximation and following the procedure used by Sodha *et al* [12] and Akhmanov *et al* [13], one can write

$$E(r,z) = A(r,z)\left[\frac{k(0)}{k(f)}\right]^{1/2} \exp[-ik(f)z], \tag{10}$$

where $k(f) = \frac{\omega}{c}[\epsilon_0'(f)]^{1/2}$ and $k(0) = \frac{\omega}{c}[\epsilon_0'(f=1)]^{1/2}$. (11)

In wave equation

$$\nabla^2 E + \frac{\omega^2}{c^2} \epsilon E = 0,$$

values of ϵ and E can be substituted form eqs. (9) and (10), which leads to parabolic equation as

$$-2ik(f)\frac{\partial A}{\partial z} + \nabla^2_\perp A + \frac{\omega^2}{c^2}\psi(r,f)A = 0. \tag{12}$$

Putting

$$A(r,z) = A_0(r,z)\exp[-i\int k(f)dS]$$

and separating real and imaginary parts, one gets

$$2\frac{\partial S}{\partial z} + \left[\frac{\partial S}{\partial r}\right]^2 = \frac{1}{k^2(f)A_0}\left[\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right.$$

$$+ \frac{\omega^2}{k^2(f)c^2}\psi(r,f) \tag{13}$$

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r} + A_0^2\left[\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right] = 0. \tag{14}$$

The solution of eqs. (13) and (14) can be written as

$$A_0^2 = \frac{E_0^2}{f^2}\exp\left[\frac{-r^2}{r_0^2 f^2}\right],$$

$$S = \frac{r^2}{2}\beta(z) + \eta(z), \tag{15}$$

$$\beta = \frac{1}{f}\frac{\partial f}{\partial z},$$

where β corresponds to the inverse radius of curvature of the wave front and $r_0 f$ is the width of the main beam in the medium.

4. Self-focusing in axial inhomogeneous plasma due to collisional non-linearity

The non-linear part of the dielectric constant of the collisional homogeneous plasma can be written as

$$\Phi(< EE >) = \omega_{p0}^2/\omega^2 W(z) \quad \left(1 + \frac{1}{2}\alpha EE^*\right)^{\frac{1}{2}s-1} \tag{16a}$$

where linear part of dielectric constant is

$$\epsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega^2} W(z). \tag{16b}$$

Here characteristic parameter α is given by

$$\alpha = \frac{e^2 M}{6k_B T_0 m^2 \omega^2}. \tag{17}$$

In eq. (16), s is a parameter characterizing the nature of collisions in plasma.

Substituting for E from eq. (10) and β, S and A_0^2 from eq. (15) in eq. (16a) and using paraxial-ray approximation, Φ can be written as

$$\Phi(< EE >) = \left(\frac{k(0)A_0^2}{2k(f)}\right) \approx \left[\frac{k(0)\alpha A_0^2}{2k(f)f^2}\right.$$

$$+ r^2\Phi' \frac{k(0)\alpha A_0^2}{2k(f)f^2} \quad r = 0. \tag{18a}$$

One knows that

$$\Phi\left[\frac{k(0)\alpha A_0^2}{2k(f)f^2}\right] = \frac{\omega_p^2}{\omega^2}W(z)\left|1 - \left\{1 + \frac{k(0)\alpha A_0^2}{2k(f)f^2}\right\}^{\frac{1}{2}s-1}\right.$$

After differentiating this with respect to r^2, one can obtain Φ' as

$$\Phi'\left[\frac{k(0)\alpha A_0^2}{k(f)f^2}\right] = -\frac{k(0)\alpha E_0^2}{2k(f)f^2}\frac{\omega_{p0}^2}{\omega^2}W(z)$$

$$\left\{1 + \frac{k(0)\alpha E_0^2}{2k(f)f^2}\right\}^{\frac{1}{2}s-2}\left(\frac{1}{2}s - 1\right)\left(-\frac{1}{r_0^2 f^2}\right).$$

Substituting this value of φ' in eq. (18a), one gets

$$\Phi(<EE>) = \Phi\left[\frac{k(0)\alpha E_0^2}{2k(f)f^2} - r^2\frac{\omega_{p0}^2}{\omega^2}W(z)\right.$$

$$\left.\frac{k(0)\alpha E_0^2}{2k(f)r_0^2 f^4}\times\left(1-\frac{1}{2}s\right)\left[1+\frac{k(0)\alpha E_0^2}{2k(f)f^2}\right]^{\frac{1}{2}s-2}\right] \quad (18b)$$

Again substituting the value of $\Phi(<EE^*>)$ from eq. (18b) in eq. (9) one gets the value of $\Psi(r,f)$ as

$$\Psi(r,f) = -r^2\frac{k(0)\alpha E_0^2}{2k(f)r_0^2 f^4}\frac{\omega_{p0}^2}{\omega^2}W(z)\left(1-\frac{1}{2}s\right)$$

$$\times\left[1+\frac{k(0)\alpha E_0^2}{2k(f)f^2}\right]^{\frac{1}{2}s-2} \quad (19)$$

Let us substitute the value of A_0^2 and S from eq. (15) and $\Psi(r,f)$ from eq. (19) in the eq. (13). Now equating the r^2 coefficients of both sides of resulting equation (following the WKB approximation) and substituting the value of β, one obtains

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2(f)r_0^4 f^3} - \left(1-\frac{1}{2}s\right)\frac{\omega_{p0}^2 W(z)}{k^2(f)c^2}\frac{k(0)\alpha E_0^2}{2k(f)r_0^2 f^3}$$

$$\left[1+\frac{k(0)\alpha E_0^2}{2k(f)f^2}\right]^{\frac{1}{2}s-2} \quad (20)$$

Above self-focusing equation has been modified and rewritten for different types of axial inhomogeneities by putting the different functions of density profile $W(z)$ as discussed in Section 3. All above modified self-focusing equations are difficult to solve analytically and thus have been solved numerically by the computer for the typical sample plasma with the following parameters $\omega = 1 \times 10^{14}$ rad s^{-1}, $\omega_p = 5.5 \times 10^{13}$ rad s^{-1}, $T_0 = 10^5$ °K, $r_0 = 30$ μm, s and -3, N_0 9 5

Figures 2a and 2b show the variation of focusing parameter (f) *versus* propagation distance (z) for different types of axially inhomogeneious plasma medium, namely, axially decreasing, increasing, exponential varying, parabolic varying *etc*., for the non-linearity introduced due to electron-neutral particle collision.

Examination of Figures 2a and 2b and Table 1 indicates that the value of $z_{min(1)}$ is almost the same (nearly 0.01) for all types of axial inhomogeneities except for exponential eq. (5b) [to be called exp. (2)] type, where it is very small

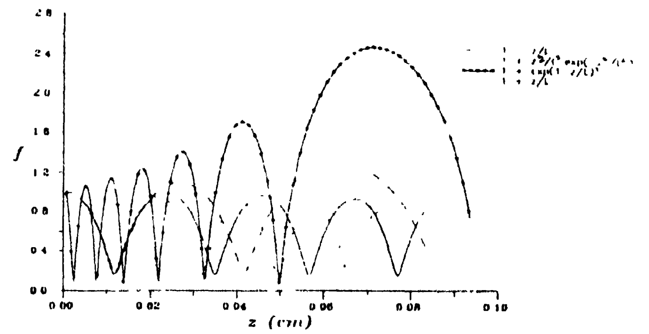

Figure 2(a). Graphs between focusing parameter (f) *versus* propagation distance (z) for electron-neutral particle collision dominated (s = 1) non-linearity in different types of axially inhomogeneous plasmas Here L = 0 2 cm

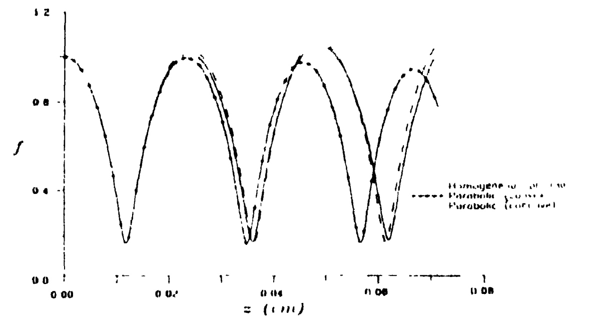

Figure 2(b). Graphs between focusing parameter (f) *versus* propagation distance (z) for s = 1 in different types of axially inhomogeneous plasmas Here L = 0 2 cm

Table 1. For different types of axially inhomogeneous plasmas and electron-neutral particle dominated non-linear interaction, minimum and maximum values of focusing parameter (f) and corresponding values of propogation distance (z) Here L = 0 2 cm

Different types of axially inhomogeneous plasma* →		Linearly increasing	Linearly decreasing	Exponential (1)	Exponential (2)	Parabolic (1) (concave)	Parabolic (2) (concave)
First minima	$f_{min(1)}$	0 1550	0 1630	0.1590	0 0770	0 1590	0 1590
	$Z_{min(1)}$	0 0117	0 0119	0 0117	0 0024	0 0117	0 0117
Second minima	$f_{min(2)}$	0 1480	0 1770	0 1560	0 0900	0 1560	0 1610
	$Z_{min(2)}$	0 0318	0 0416	0 0348	0.0075	0 0348	0.0359
First maxima	$f_{max(1)}$	0 9330	1 1030	0 9900	1 0590	0 9900	1 0090
	$Z_{max(1)}$	0 0219	0 0264	0 0234	0 0048	0 0234	0 0237
Second maxima	$f_{max(2)}$	0 8840	1 3090	0.9660	1 1390	0 9640	1.0420
	$Z_{max(2)}$	0 0408	0 0629	0 0458	0.0107	0 0458	0 0489

*Note . Ref eqs (3), (4), (5a), (5b), (6a), (6b) for inhomogeneous functions (nearly 0.0024). Similarly the value of $z_{min(2)}$ for linearly decreasing type inhomogeneity is little higher than linearly increasing exp (1) [represented by exponential eq. (5a)], parabolic (1), parabolic (2) [see eqs. (6a) and (6b)] type axial inhomogeneities. For exp (2) type of inhomogeneity, separation between successive increasing number of maximas increases rapidly with increasing value of propagation distance and value of f_{max} continuously increases with the increasing value of z and attains value much higher than 1. Value of successive f_{max} also increases, even at much faster rate, in case of linearly decreasing inhomogeneous plasma for collisional non-linearity. Result of present analysis indicates that in case of linearly decreasing inhomogeneous plasma for $z > 0.086$, value of f continuously increases and attains saturation value *i.e.* no oscillatory behaviour in f *versus* z is observed after $z = 0.086$.

Figures 3a and 3b show the dimensionless beam width parameter (f) as a function of z for various types of axially inhomogeneous plasma and for electron-ion collision non-linearity ($s = -3$). Plots are shown for $z < L$ (here value of inhomogeneity scale length (L) is 0.2 cm) because for some of the inhomogeneity considered here, plasma density becomes imaginary and unrealistic when $z > L$. Results indicate that computer generated programme fails to plot variation in such cases for $z > L$, as expected. Comparison

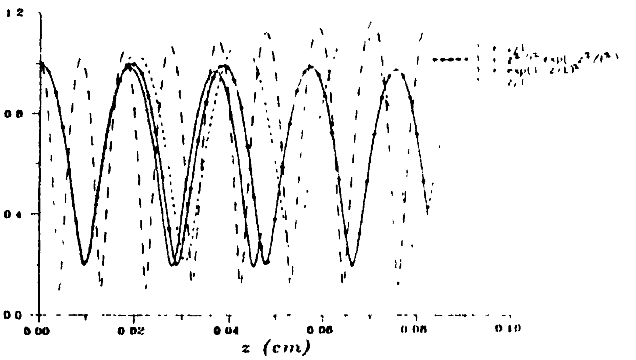

Figure 3(a). Graphs between focusing parameter (f) *versus* propagation distance (z) for electron-ion collision dominated ($s = -3$) non-linearity in different types of axially inhomogeneous plasmas Here $L = 0.2$ cm

of these results with $s = 1$ case, indicates that although again oscillatory behaviour in self-focusing is observed but self-focusing is less prominent here as compared to $s = 1$ case,

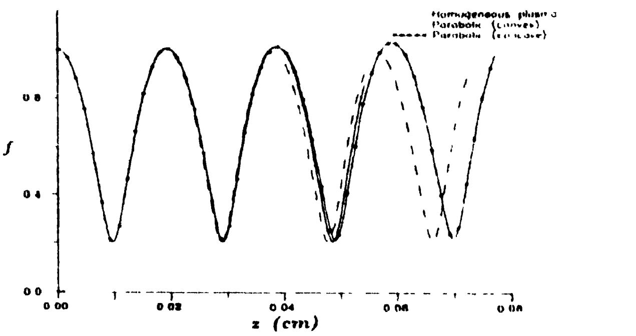

Figure 3(b). Graphs between focusing parameter (f) *versus* propagation distance (z) for $s = -3$ in different types of collisional non-linearity and in different types of axially inhomogeneous plasmas Here $L = 0.2$ cm

which is reflected in the values of f_{min} and f_{max} (see Tables 1 and 2). Value of $z_{min(1)}$ corresponding to first minimum/first focusing point ($z_{min(1)} = 0.0096$ cm) is almost same for all type of inhomogeneities except for exp (2) type inhomogeneity. It is observed that as the value of L *i.e.* characteristic scale length of axially inhomogeneous plasma increases to very high value, the results of self-focusing for all types of inhomogeneities considered in the present analysis, tend to the results of homogeneous plasma except for exp (2) type inhomogeneity. Results for $L = 200$ cm are presented in Table 3.

In Figures 4 and 5, plots of f *versus* z for exponentially, axially inhomogeneous plasma are shown. Here, value of inhomogeneity scale length (L) is 0.12 cm and oscillatory behaviour of f is observed even for $z > L$. Results in Figure 4 for collisional non-linearity $s = -3$ and exp (2) type inhomogeneity indicate that the values of f_{max} initially increases up to $f_{max} = 1.28$ for $z = 0.206$ cm and then start decreasing slowly with the increasing value of z. In this type of inhomogeneous plasma considered here for analysis, the variation in electron charge density with z is responsible for this behaviour. The position (*i.e.* value of z) of maximum

Table 2. For different types of axially inhomogeneous plasmas and electron-ion dominated non-linear interaction, minimum and maximum values of f and corresponding values of z Here $L = 0.2$ cm

Different types of axially inhomogeneous plasma →		Linearly increasing	Linearly decreasing	Exponential (1)	Exponential (2)	Parabolic (1) (concave)	Parabolic (2) (concave)
First minima	$f_{min(1)}$	0.1980	0.2040	0.2010	0.0900	0.2010	0.2010
	$Z_{min(1)}$	0.0096	0.0098	0.0096	0.0042	0.0096	0.0096
Second minima	$f_{min(2)}$	0.1920	0.2100	0.2010	0.0900	0.2010	0.2030
	$Z_{min(2)}$	0.0279	0.0306	0.0291	0.0131	0.0291	0.0291
First maxima	$f_{max(1)}$	0.9790	1.0340	0.9960	1.0220	0.9960	1.0090
	$Z_{max(1)}$	0.0188	0.0200	0.0192	0.0087	0.0192	0.0195
Second maxima	$f_{max(2)}$	0.9630	1.0790	0.9890	1.0440	0.9880	1.0120
	$Z_{max(2)}$	0.0366	0.0419	0.0383	0.0176	0.0383	0.0393

Table 3. For different types of axially inhomogeneous plasmas and electron-neutral particle dominated non-linear interaction, minimum and maximum values of f and corresponding values of z. Here $L = 200$ cm

Different type of axially inhomogeneous plasma	First minima		Second minima		First maxima		Second maxima	
	$f_{min(1)}$	$z_{min(1)}$	$f_{min(2)}$	$z_{min(2)}$	$f_{max(1)}$	$z_{max(1)}$	$f_{max(2)}$	$z_{max(2)}$
Linearly increasing	0 159	0117	0 158	0 0354	0.999	0237	0.999	0471
Linearly decreasing	0 159	0117	0 158	0 0354	0 999	0237	0 999	0471
Exponential (1)	0 159	0117	0 158	0 0354	0 999	0237	0.999	.0471
Exponential (2)	0 101	0024	0 054	0 0069	0 996	0045	0 990	.0089
Parabolic (1) (convex)	0 159	0117	0 158	0 0035	0 999	0237	0.999	.0471
Parabolic (2) (concave)	0 159	0117	0 158	0 0354	0 999	0237	0 999	0471
Homogeneous	0 159	0117	0 159	0 0363	1 026	0240	1 055	0489

value of f_{max} depends upon the chosen value of characteristic scale length of axially inhomogeneity. Depth of focusing

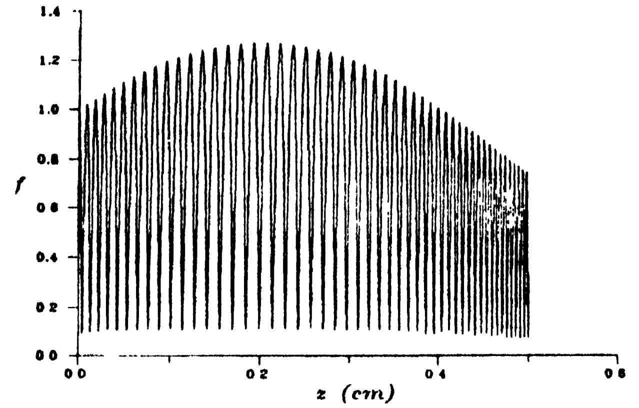

Figure 4. Variation of focusing parameter (f) *versus* propagation distance (z) for exponential ($1 + \exp(1 - z/L)^2$) type axially inhomogeneous plasma with $L = 0.2$ cm and $s = -3$

i.e. f_{min}, is also found to follow density profile functions [see Figure 1].

Results for exp (1) type inhomogeneity, where electron charge density first increases and after attaining maximum value starts decreasing with z, are shown in Figure 5.

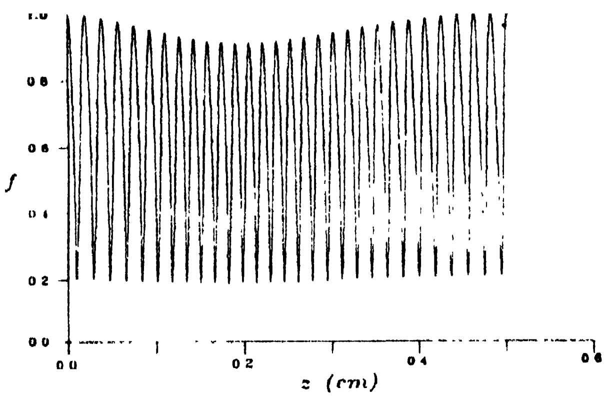

Figure 5. Variation of focusing parameter (f) *versus* propagation distance (z) for exponential ($1 + z^2/L^2 \exp(z^2/L^2)$) type axially inhomogenous plasma with $L = 0.2$ cm and $s = 3$

Comparison of these results with exp (2), indicates that here f_{max} first decreases to 0.909 (minimum value) at $z = 0.206$ cm and then starts increasing continuously just opposite to the results of exp (1) type inhomogeneity.

5. Discussion

In this paper, results of self-focusing study of linearly polarised wave with a Gaussian initial intensity profile, based on steady state non-linear refraction theory in an inhomogeneous transparent plasma in the aberration-less paraxial approximation, are presented. Intense electromagnetic beam with non-uniform intensity distribution along its wave-front causes various non-linear phenomenon while propagating through plasma. Electrons in plasma are heated non-uniformly and redistribution of charges takes place. Different types of collision processes between different types of particles present in plasma such as electron-neutral particle collision, electron-ion collision *etc*. play an important role and contribute differently to non-linear behaviour.

A non-linear variation in the dielectric constant of plasma along the wave-front is set up because of collision-associated redistribution of charges. The combined effect of both the non-linearity induced in the transverse direction due to propagating intense laser beam and the plasma inhomogeneity in axial direction, has been considered. Equation of self-focusing for different situations (*i.e.* for different types of axial inhomogeneities) have been obtained and solved numerically with the help of computers. Plot of beam width parameter (f) as a function of propagation distance has been drawn.

The non-linear refraction of electromagnetic beams has been studied theoretically by many investigators for dielectric medium [12] and homogeneous plasma [13] *etc*. In the present study, the plasma medium is considered to be axially inhomogeneous and is close to more realistic situation.

In all types of axially inhomogeneous plasmas considered in the present study, initially up to some degree of penetration, self-focusing behaviour shows oscillatory nature. Just on entering the plasma, laser beam aperture starts decreasing and after reaching minimum value (different for different types of plasmas and for different types of non-linearties responsible for self-focusing) at focusing (*i.e.* for f_{min}), starts increasing. Due to small value of beam aperture, diffraction effect starts dominating over focusing effect and beam starts defocusing *i.e* value of f starts increasing after reaching a minimum value. When beam-spread is large, contribution due to diffraction effect is counter-balanced or in fact, dominated by self-focusing effect due to various types of non-linearities. Beam again starts focusing *i.e.* value of f starts decreasing after reaching f_{max} value. This process repeats again and again giving oscillatory nature to beam aperture. Because of inhomogeneous nature of plasma, repetitive nature is not exactly the same as observed in case of homogeneous plasma.

For different types of axial inhomogeneities, values of $f_{min(1)}, f_{min(2)}$ ... and $f_{max(1)}, f_{max(2)}$ ... are different (see results in Table 1). In case of linearly increasing exp(1) and convex type parabolic axial inhomogeneous plasmas, the value of f_{max} with z is found to decrease while in case of linearly decreasing exp(2) and concave type parabolic axially inhomogeneous plasmas, it increases with z. In case of linearly increasing exp(1) and convex type parabolic axial inhomogeneous plasmas, due to change in charge density, plasma frequency increases with z. This causes the increase in the non-linear part of self-focusing. Hence, f cannot reach upto 1 (*i e* beam aperture even at f_{max} is less than initial value at $z = 0$) and it starts focusing before reaching $f = 1$. These results are quite different than reported by various investigations for homogeneous plasma [13].

In case of linearly increasing inhomogeneous plasma, value of $f_{min(1)}$ is slightly larger than $f_{min(2)}$ whereas $f_{max(2)}$ is slightly less than $f_{max(1)}$. Careful analysis of Figure 2 indicates that value of f_{min} decreases with increasing order of minima *i e* focusing effect is much stronger at higher penetration in the medium.

In case of linearly decreasing axially inhomogeneous plasma, beam initially focuses, then defocuses and again focuses, this pattern continues up to some extent beyond which defocusing of the beam prevails. This behaviour is not common to other type of inhomogeneous mediums considered in the present study. In this type of inhomogeneous medium, as the beam propagates, charge density decreases causing ω_p to decrease. Beyond some distance, due to decrease in ω_p, minimum value of beam power for self-focusing increases *i e.* here the critical power is a function of propagation distance. Now at a certain value of z, incident beam power level becomes less than critical power level and hence beam does not focus. Apart from that for $z > L$, charge density function for linearly decreasing axially inhomogeneous plasma [eq. (4)] becomes negative, leading to purely imaginary plasma frequency. This may also contribute to the observed non-oscillatory behaviour of beam at deeper penetration in medium.

The present analysis of self-focusing in inhomogeneous plasma leads to some very interesting information. It is observed that self-focusing behaviour of the propagating beam is a complex function of inhomogeneity and very strongly depends on the density function. Axial inhomogeneity plays an important role and focusing length depends upon the nature of axial inhomogeneity.

In case of linearly increasing and parabolic convex type axially inhomogeneous mediums, observed oscillatory behaviour appears to be damped *i e* the value of oscillating f_{max} decreases continuously with z. This damping is not due to any kind of loss mechanism but due to charge density variations.

In case of laser induced fusion process, the nature and characteristic of plasma near the pellet is going to play a very important role in surface heating of the fusion pellet. The knowledge of the plasma density profile which will be created across the pellet is essential to assess the laser energy density function at pellet

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References

[1] S A Akhmanov, A P Sukhorukov and R V Khokhlov *Sov Phys JETP* **23** 1025 (1996)

[2] V N Lugovoi and A M Prokhorov *Progress in Lasers and Laser Fusion* eds Arnold Perlumter and M Susan Widmayer (New York Plenum) 309 (1975)

[3] V V Korobkin and M Ya Romanovsky *Phys Rev* **E49** 2316 (1994)

[4] K Fldman and R Sigel *Phys Rev Lett* **34** 799 (1975)

[5] A L Dyshko V N Lugovoi and A M Prokhorov *JETP Lett* **6** 146 (1967)

[6] Bendow Bernard, D Garmino Peter and Izvar Narikis *J Opt Soc Am* **17** 656 (1981)

[7] M S Sodha, S Konar and K P Maheshwari *J Plasma Phys* **48** 107 (1992)

[8] M S Sodha, A K Ghatak and V K Tripathi *Progress in Optics* **Vol. XIII** (Amsterdam North-Holland) 169 (1975)

[9] D P Garuchava, Z I Rostomashvili and N I Tsintsadze *Sov J Plasma Phys* **12** 776 (1986)

[10] M S Sodha, S Koran and K P Maheshwari *Indian J Pure Appl Phys* **32** 660 (1994)

[11] R K Khanna and K Baheti *Asian J Phys* **7** (1999)

[12] M S Sodha, A K Ghatak and V K Tripathi *Self Focusing of Laser Beam in Dielectrics, Plasmas and Semiconductors* (New Delhi Tata McGraw Hill) (1974)

[13] S A Akhmanov, A P Sukhorukov and R V Khokhlov *Sov Phys Uspekshi* **10** 609 (1968)